Uniaxial Plate Stretching

Setup and assumptions

A thin rectangular plate of length a, width b and thickness $t \ll a, b$ is loaded in its plane by a uniform tensile traction on the two edges $x = \pm a/2$, of resultant F, so the applied stress is $\sigma_0 = F/(b\,t)$. The side edges $y = \pm b/2$ and both faces are traction-free. The material is linear elastic and isotropic (E, ν); strains are small.

Plane stress

Because the plate is thin and its faces are free, the through-thickness stresses cannot build up: $\sigma_{zz} = \tau_{xz} = \tau_{yz} \approx 0$. Hooke's law then reduces to the plane-stress relations

$$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{E}, \qquad \varepsilon_y = \frac{\sigma_y - \nu\sigma_x}{E}, \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}, \qquad \varepsilon_z = -\frac{\nu\,(\sigma_x + \sigma_y)}{E}. \tag{1}$$

Note $\varepsilon_z \neq 0$: the plate is free to thin. (Plane *strain*, where $\varepsilon_z = 0$ is enforced, is a different idealisation.)

The homogeneous field solves the problem exactly

Try the uniform state

$$\sigma_x = \sigma_0, \qquad \sigma_y = 0, \qquad \tau_{xy} = 0 \qquad \text{everywhere.} \tag{2}$$

Check it against everything the exact 2D theory requires:

- *Equilibrium* $\partial\sigma_x/\partial x + \partial\tau_{xy}/\partial y = 0$ and its companion: satisfied trivially, all fields constant.
- *Boundary conditions*: on $x = \pm a/2$ the traction is $\sigma_x = \sigma_0$ as applied; on $y = \pm b/2$ the traction components σ_y and τ_{xy} vanish as required. Satisfied *pointwise*, not just on average.
- *Compatibility*: the strains are constant, so a displacement field exists; explicitly, measuring from the plate centre, $u = \varepsilon_x\,x$ and $v = \varepsilon_y\,y$.

By uniqueness of linear elastic solutions, this *is* the solution — no approximation involved.

Results

$$\boxed{\varepsilon_x = \frac{\sigma_0}{E}, \qquad \varepsilon_y = -\nu\,\varepsilon_x, \qquad \varepsilon_z = -\nu\,\varepsilon_x} \qquad \Rightarrow \qquad \Delta a = \varepsilon_x\,a, \qquad \Delta b = \varepsilon_y\,b, \qquad \Delta t = \varepsilon_z\,t. \tag{3}$$

The plate stretches along the load and contracts both across it and through its thickness; Poisson's ratio is the whole story of the lateral response. At $\nu = 0$ (cork-like) there is no contraction; at $\nu = 1/2$ (rubber, incompressible) the volume change $\varepsilon_x + \varepsilon_y + \varepsilon_z = (1 - 2\nu)\,\varepsilon_x$ vanishes.

Worked example (simulator defaults)

$E = 10$ MPa, $\nu = 0.3$, $a = 500$ mm, $b = 250$ mm, $t = 2$ mm, $F = 400$ N:

$$\sigma_0 = \frac{400}{0.25 \times 0.002} = 0.8\text{ MPa}, \qquad \varepsilon_x = 8\%, \qquad \varepsilon_y = -2.4\%, \tag{4}$$

so $\Delta a = 40$ mm, $\Delta b = -6$ mm and $\Delta t = -0.048$ mm. Set the simulator sliders to these values and compare; then sweep ν from 0 to 0.5 and watch only the lateral response change.